Non-coplanar beam orientation optimization for total marrow irradiation using IMRT

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Bone Marrow Transplants (BMTs)

- Method of treatment for blood and bone marrow cancers (leukemia and lymphoma); also for aplastic anemia and sickle cell disease
- To transplant bone marrow, existing bone marrow must be eradicated – current method is **total body irradiation** (TBI)
- More effective methods of bone marrow elimination – **total marrow irradiation** (TMI)

Intensity Modulated Radiation Therapy (IMRT)

- Conventional radiotherapy: beam is of homogeneous intensity
- IMRT: beam is broken into numerous beamlets; each beamlet's intensity is controllable
- Used successfully for other types of cancer (head-and-neck)

Has IMRT been applied to TMI before?

Yes, but

- Not within a mathematical framework
- Previous work studies irradiation from a greater distance

... So what are the problems?

- For a set of beam directions, which beamlet intensities are optimal? And...
- Which beam directions do we use in the first place?

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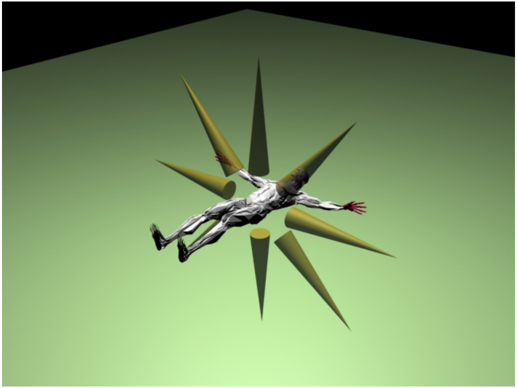
Beam Orientation Optimization (BOO)

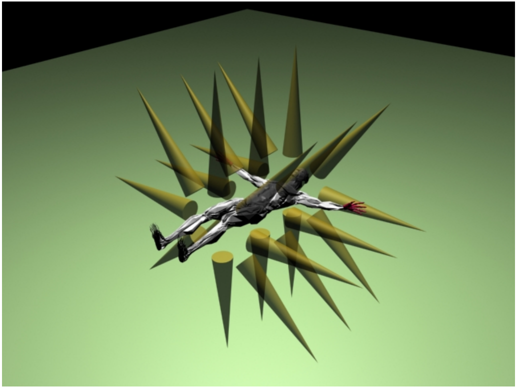
- Beams are obtained by
  - Rotating the gantry – 10° increments
  - Translating the couch along patient's vertical axis – 10 cm increments
- Beams are **non-coplanar**

Coplanar vs. non-coplanar



Coplanar

Coplanar vs. non-coplanar



Non-coplanar

BOO Model Overview

- Set of beam orientations is denoted by Θ
- Quality of set is denoted by function $\mathcal{F}(\Theta)$
- Goal is to optimize $\mathcal{F}(\Theta)$ over all possible sets of beams

The function $\mathcal{F}$

- Beam set quality can be formulated in many ways
- We use fluence map optimization (FMO) value from Romeijn et al. [2006]

FMO Model Overview (1)

- *For a set of beam directions, which beamlet intensities are optimal?*
- x_i is intensity of beamlet i, $i \in B_\Theta$
- z_{js} is dose in voxel j in structure s, $j \in \{1, \ldots v_s\}$, $s \in S$
- $z_{js} = \sum_{i \in B_\Theta} D_{ijs} x_i$

FMO Model Overview (2)

- The dose in each voxel can be penalized:

$$F_{js}(z_{js}) = \left[\underline{w}_s \left(T_s - z_{js} \right)_+^{\underline{p}_s} + \overline{w}_s \left(z_{js} - T_s \right)_+^{\overline{p}_s} \right] \qquad (1)$$

- $\underline{w}_s$, $\overline{w}_s$ are weights for underdosing and overdosing; $\underline{p}_s$, $\overline{p}_s$ are powers for underdosing and overdosing
- Total penalty is $\sum_{s \in S} \left(\frac{1}{v_s} \sum_{j=1}^{v_s} F_{js}(z_{js}) \right)$

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FMO Model Overview (3)

Model is

$$
\begin{aligned}
\min \quad & \sum_{s \in S} \left(\frac{1}{v_s} \sum_{j=1}^{v_s} F_{js}(z_{js}) \right) \\
\text{subject to} \quad & z_{js} = \sum_{i \in B_\Theta} D_{ijs} x_i, \ \forall j \in \{1, \ldots, v_s\}, \ s \in S \\
& x_i \geq 0, \ \forall i \in B_\Theta
\end{aligned}
\tag{2}
$$

FMO model is convex, solvable using projected gradient method

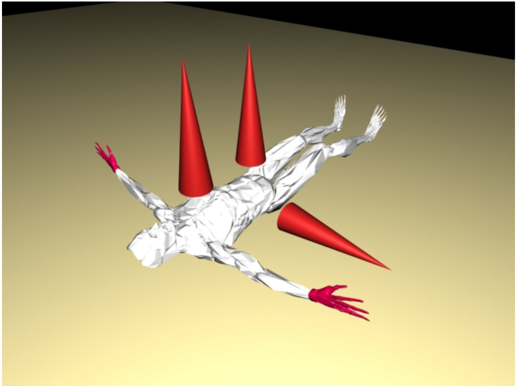
Add/Drop (A/D) Algorithm

- Neighborhood search algorithm
- Previously used in Kumar [2005] and Aleman et al. [2008]

One iteration of A/D

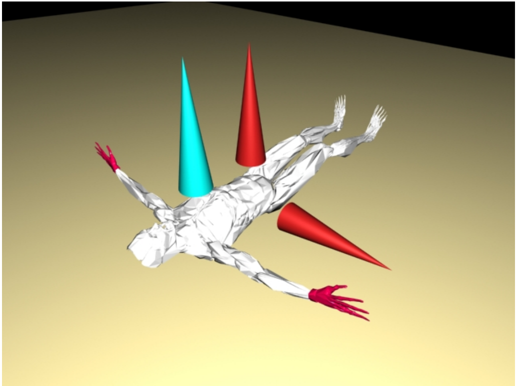
1. Select a beam and a component (gantry angle/z trans.)
2. Enumerate all the Θ in the beam-component neighborhood of current Θ ($\Theta^{(i)}$)
3. Calculate $\mathcal{F}$ for all of these solutions
4. Set $\Theta^{(i+1)}$ to the most improving new Θ (if any)

One iteration of A/D - graphical example



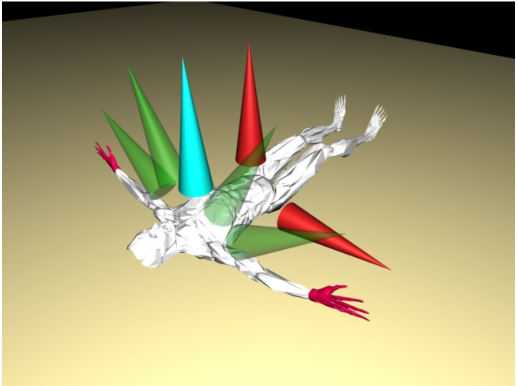
Current solution – $\Theta^{(i)}$, $\mathcal{F}(\Theta^{(i)}) = 700$

One iteration of A/D - graphical example



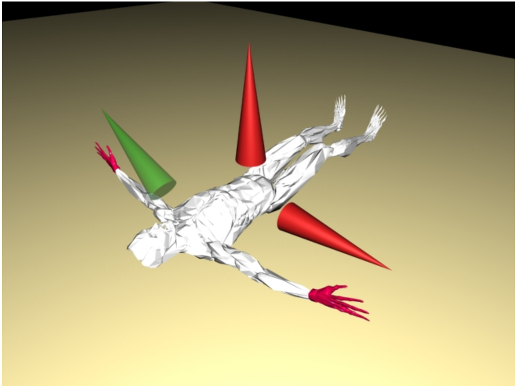
Select a beam and a component

One iteration of A/D - graphical example



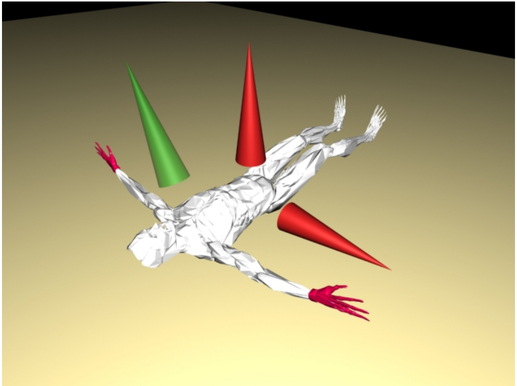
Enumerate solutions in the neighborhood of $\Theta^{(i)}$

One iteration of A/D - graphical example



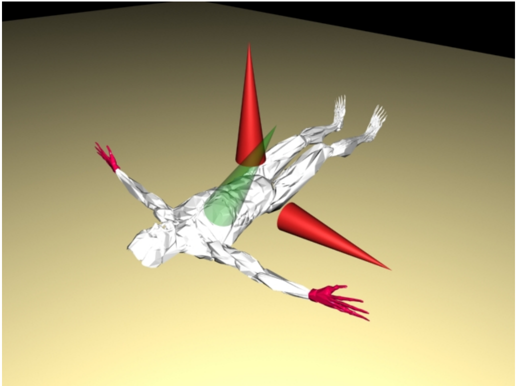
$$\Theta_1,\ \mathcal{F}(\Theta_1) = 490$$

One iteration of A/D - graphical example



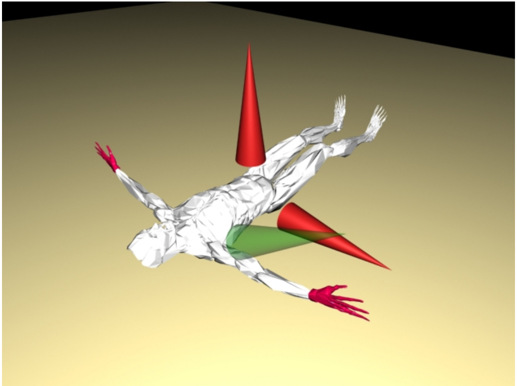
Θ_2, $\mathcal{F}(\Theta_2) = 520$

One iteration of A/D - graphical example



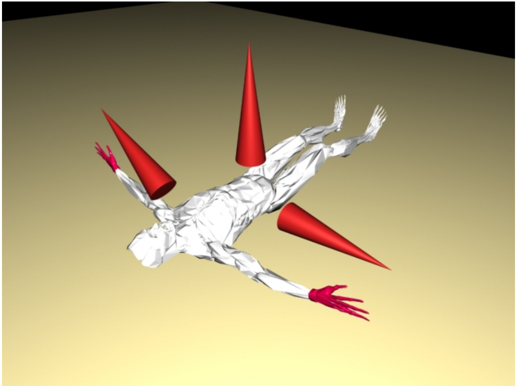
Θ_3, $\mathcal{F}(\Theta_3) = 780$

One iteration of A/D - graphical example



$$\Theta_4,\ \mathcal{F}(\Theta_4) = 770$$

One iteration of A/D - graphical example



Θ_1 is most improving solution $\rightarrow$ set $\Theta^{(i+1)} = \Theta_1$

Types of A/D

- Most basic is sequential – (1, G), (1, z), (2, G), (2, z) ...
- Other types: probabilistic, dynamic δ

Probabilistic A/D

- In each iteration, beam and component pair are randomly selected
- Selection probabilities calculated using the average of the recent improvements of each pair, relative to overall average

$$\bar{p}\left(b,d\right) = \Pr\left(\mathbf{B} = b, \mathbf{C} = d\right) = \frac{1}{k|D|} + \frac{\alpha}{k|D|}\left(\frac{\Delta_{bdr} - \bar{\Delta}_m}{\bar{\Delta}_m}\right) \quad (3)$$

- Sensitivity of probabilities to recent improvement can be controlled using an additional parameter α

Dynamic δ

- In basic A/D, neighborhood size (δ) is constant
- In dynamic δ A/D, δ changes to reflect how much solution moved previously
- Large moves $\rightarrow$ large neighborhood sizes
- Small moves $\rightarrow$ small neighborhood sizes

$$\bar{\delta_G} = c_{zG}||\bar{\Theta} - \Theta^{(i)}|| \tag{4}$$

$$\bar{\delta_z} = c_{Gz}||\bar{\Theta} - \Theta^{(i)}|| \tag{5}$$

Results

- Executed on Beowulf CentOS cluster with 32 nodes
- Each node has 8 2GHz processors and 8GB of memory
- In each A/D iteration, each FMO calculation assigned to a different processor

Numerical Results

10 executions, 12 hours, $\delta_G = 20°$, $\delta_z = 20$ cm, random starting point

A/D Type	Avg. Final FMO	Avg. Num. Iter.s	Avg. Prop. Improv. Iter.s
SC	11370.4	40.9	32.69%
Pr, $\alpha = 0$	10978.5	42.7	35.92%
Pr, $\alpha = 0.25$	11429.1	41.5	35.17%
Pr, $\alpha = 0.5$	11083.4	45.5	35.95%
Pr, $\alpha = 0.75$	11133.8	44.2	33.89%
Pr, $\alpha = 1$	11071.2	48.5	37.21%
Dδ, $c_{Gz} = 2.5$, $c_{zG} = 1$	10942.5	30.1	36.97%

Treatment Plan Criteria

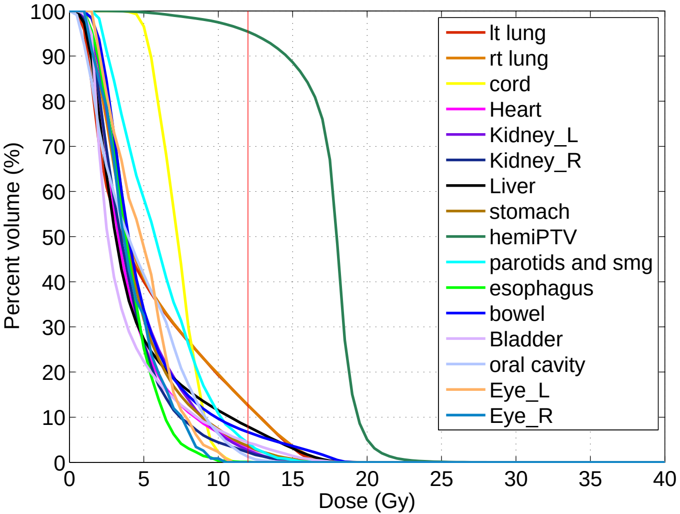
Criteria used:

- 95% of bone marrow receives more than 12Gy
- At most 20% of bone marrow receives more than 20Gy
- 0% of bone marrow receives more than 25Gy
- Majority of OAR volume receives less than 8Gy

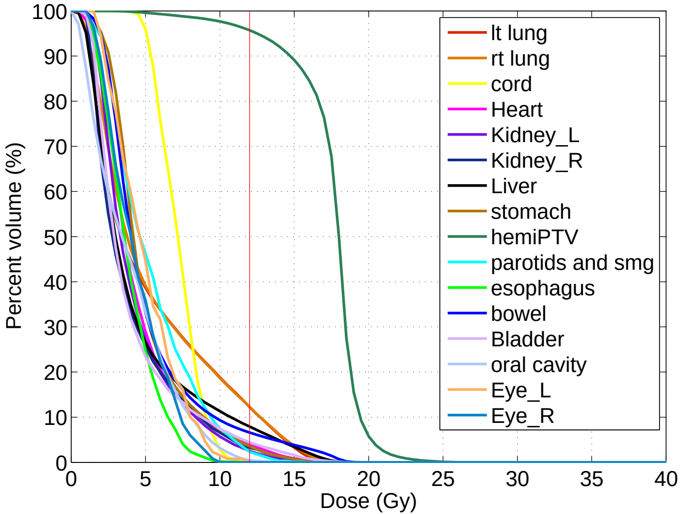
General Results

- Generally all criteria were met
- Hardest to spare well: spinal cord and lungs

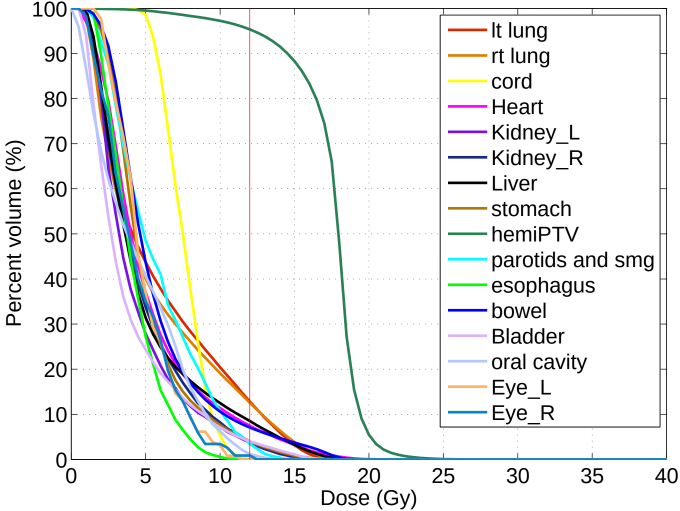
DVHs – Sequential Cycling A/D, 30 beams



DVHs – Probabilistic A/D ($\alpha = 0.75$), 30 beams



DVHs – Dynamic δ A/D, 30 beams



Conclusions

- IMRT with non-coplanar beams allows for more effective treatment than TBI
- Add/Drop can obtain such plans in a clinically realistic timeframe and environment

The Future

- Other variants of A/D
- Other algorithms
- Clinical realizability
  - Incorporation of patient and organ motion
  - Design of arc therapy plans

Thank you for listening

Questions?

D. M. Aleman, A. Kumar, R K. Ahuja, H. E. Romeijn, and J. F. Dempsey. Neighborhood search approaches to beam orientation optimization in intensity modulated radiation therapy treatment planning. *Journal of Global Optimization*, 42(4):587–607, 2008.

A. Kumar. *Novel methods for intensity-modulated radiation therapy treatment planning*. PhD thesis, University of Florida, 2005.

H. E. Romeijn, R. K. Ahuja, J. F. Dempsey, and A. Kumar. A new linear programming approach to radiation therapy treatment planning problems. *Operations Research*, 54(2):201–216, 2006.